Influence of ion motion in the inverse Faraday effect

Abraham C.-L. Chian

Insitituto de Pesquisas Espaciais, Conselho Nacional de Desenvolvimento Científico e Tecnológico, 12200 São José dos Campos, SP, Brazil

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It is shown that ion motion can significantly affect the magnetic field generated by an intense circularly polarized electromagnetic wave in a plasma. For extremely strong waves, the magnetic field induced by the electron current in an electron-ion plasma can be nearly canceled by the ion current.

Recently, there has been considerable interest in the spontaneous generation of magnetic fields in plasma under the action of an intense electromagnetic field. A great variety of physical mechanisms have been proposed to account for the appearance of such magnetic fields.[1] In this note, we consider the inverse Faraday effect which refers to the generation of magnetic fields by a circularly polarized electromagnetic wave.

In the presence of a circularly polarized electromagnetic wave, the charged particles gyrate in circular orbits. This gyration of the charged particles induces a magnetic field along the direction of wave propagation. The inverse Faraday effect has been theoretically formulated and experimentally demonstrated in solids[2,3] and plasma.[4-6] The theoretical analysis of Steiger and Woods[6] indicates that induced magnetic fields of mega-Gauss range are possible with the lasers available today. Their treatment is relativistic, but ion dynamics is ignored. It is the aim of this note to investigate the influence of ion motion in the inverse Faraday effect.

We assume the plasma to consist of electrons $(m_e, -e)$ and ions (m_i, e). For a circularly polarized wave propagating in the z direction, the wave fields are

$$\begin{aligned} \mathbf{E} &= E_0(\cos\omega\theta, \mp \sin\omega\theta, 0)\,, \\ \mathbf{B} &= B_0(\pm \sin\omega\theta, \cos\omega\theta, 0)\,, \end{aligned} \tag{1}$$

where $\theta = t - nz/c$, n = index of refraction, $B_0 = nE_0$, and the alternate signs refer to right- and left-hand polarization, respectively. Under the action of the wave fields (1), the electrons and ions acquire the velocities

$$\begin{aligned} \mathbf{v}_e &= v_e(\pm \sin\omega\theta, \cos\omega\theta, 0)\,, \\ \mathbf{v}_i &= v_i(\pm \sin\omega\theta, \cos\omega\theta, 0)\,. \end{aligned} \tag{2}$$

Each plasma particle gyrating according to Eq. (2) contributes a magnetic dipole moment

$$\boldsymbol{\mu}_\alpha = \pm\,(q_\alpha v_\alpha^2/2\omega c)\hat{z}\,, \tag{3}$$

where $\alpha = (e, i)$, q is the particle charge, and $\hat{z} = (0, 0, 1)$. The induced magnetic field, due to the orbital motion of the plasma particles, is then

$$\mathbf{B}_{\text{ind}} = 4\pi \sum_\alpha N_\alpha \boldsymbol{\mu}_\alpha = \pm \frac{2\pi Ne}{\omega c}\,(v_i^2 - v_e^2)\hat{z}\,, \tag{4}$$

where the number density N is constant for transverse circularly polarized waves $(N_e = N_i = N)$.[7,8] Equation (4) shows that the induced magnetic field for right-circular polarization is antiparallel and for left-circular polarization is parallel to the direction of wave propagation.

It is evident, from Eq. (2), that v_α^2 and $\gamma_\alpha = (1 - v_\alpha^2/c^2)^{-1/2}$ are constant. Hence, the induced magnetic field (4) is a static field. Substitution of Eqs. (1) and (2) into the relativistic equation of motion, written in terms of the phase $\theta(\partial/\partial t \to d/d\theta,\ \mathbf{v}_\alpha \cdot \nabla \to 0)$ (Ref. 7),

$$m_\alpha \gamma_\alpha d\mathbf{v}_\alpha/d\theta = q_\alpha\left[\mathbf{E} + (1/c)\mathbf{v}_\alpha \times (\mathbf{B} + \mathbf{B}_{\text{ind}})\right]\,, \tag{5}$$

shows that

$$v_e^2 = \left(\frac{c\nu_E}{\gamma_e \pm \nu_B}\right)^2, \quad v_i^2 = \left(\frac{c\mu\nu_E}{\gamma_i \mp \mu\nu_B}\right)^2, \tag{6}$$

where $\mu = m_e/m_i$, $\nu_E = eE_0/m_e\omega c$, and $\nu_B = eB_{\text{ind}}/m_e\omega c$. Equation (4) can be rewritten as

$$\nu_B = \pm \frac{\omega_{pe}^2}{2\omega^2}\left(\frac{v_i^2}{c^2} - \frac{v_e^2}{c^2}\right)\hat{z}\,, \tag{7}$$

where $\omega_{pe}^2 = 4\pi Ne^2/m_e$. Therefore, for given ω_{pe}, ω, and μ, Eqs. (6) and (7) yield a relation between the induced magnetic field and the amplitude of the driving electromagnetic wave.

The wave strength parameter ν_E is related to the intensity I_0 of the electromagnetic wave by

$$\nu_E = \frac{e}{m_e\omega c}\left(\frac{4\pi I_0}{nc}\right)^{1/2}, \tag{8}$$

where $I_0 = (c/4)\,|\mathbf{E}\times\mathbf{B}| = ncE_0^2/4\pi$. By use of Maxwell's equations, the dispersion relation is found to be[8]

$$n^2 = 1 - \left(\frac{1}{\gamma_e \mp \nu_B} + \frac{\mu}{\gamma_i \pm \mu\nu_B}\right)\frac{\omega_{pe}^2}{\omega^2}\,. \tag{9}$$

Thus, Eqs. (6)–(9) can be solved simultaneously to show the dependence of the induced magnetic field on other parameters of the system.

Figure 1 displays some computed solutions of the induced magnetic field strength ν_B as a function of the wave strength ν_E. For the case of electron plasma[6] $(\mu = 0)$, ν_B increases monotonically with ν_E, and approaches the maximum value $\omega_{pe}^2/2\omega^2$ asymptotically, as can be deduced from Eq. (7) by setting $v_i = 0$. We can see, from Fig. 1, that the effect of ion motion is to reduce the magnitude of the induced magnetic field. In fact, the inclusion of ion motion shows that ν_B does not always increase monotonically with ν_E; rather, after reaching a certain maximum value, ν_B then decreases with increasing ν_E. For extremely large wave amplitudes, the magnetic field induced by the electron current can be nearly canceled by the ion current; since in the presence of extremely strong waves, both v_e and v_i

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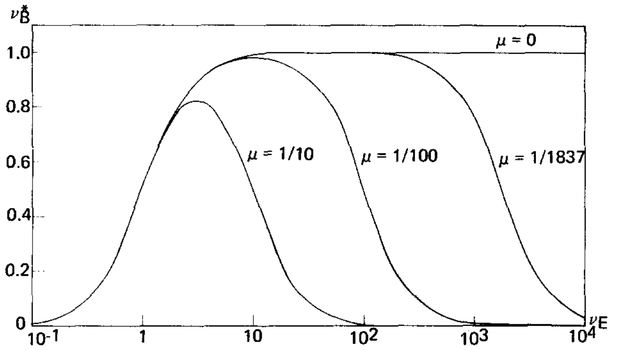

FIG. 1. Variation of $\nu_B^* \equiv (\omega_{pe}^2/2\omega^2)^{-1}\nu_B$, with ν_E for indicated values of μ; $\omega_{pe}^2/\omega^2 = 1/5$.

are driven relativistic, and Eq. (7) shows that $\nu_B \to 0$ as v_i, $v_e \to c$. The influence of ion motion in the inverse Faraday effect is more pronounced for higher values of the mass ratio μ, as shown in Fig. 1. For the case of electron-positron plasma ($\mu = 1$), Eq. (6) shows that apart from a small difference (the second term in the denominators) due to the opposite orientation of the $q_\alpha \mathbf{v}_\alpha \times \mathbf{B}_{\mathrm{ind}}$ force for electrons and positrons, v_e^2 is essentially the same as v_i^2; thus, the inverse Faraday effect nearly vanishes for all wave amplitudes.

In conclusion, we note that the cold plasma results can be straightforwardly extended to a finite temperature plasma[8] by replacing m_α by a temperature corrected mass $m_\alpha^* = m_\alpha + (KT_\alpha/c^2)\Gamma_\alpha/(\Gamma_\alpha - 1)$, where Γ_α is the "adiabatic" constant. Moreover, the effect of plasma streaming can easily be obtained from the stationary plasma results through a Lorentz transformation.[7] Finally, the inverse Faraday effect in plasma should be relevant over time scales shorter than twice the oscillation period of the driving wave, beyond which the wave becomes unstable and the parametric instability develops.[9,10]

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Multi-channel ion beam overlap

Thomas P. Wright

Sandia National Laboratories, Albuquerque, New Mexico 87185
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Calculations of ion flux enhancement from overlap of many plasma-channel-transported beams onto inertial confinement fusion targets give factors of 10 with efficiencies around 85%. Scaling relations are presented for different beam parameters and overlap geometries.

Continuing progress in light-ion beam technology is leading to promising schemes for achieving ion-beam driven inertial confinement fusion.[1-3] The theory and geometry of multiple beam overlap from channel transport of intense particle beams[4,5] is used to investigate light ion overlap for realistic beam parameters in this note. Multiple "disks" of beams arranged on conical surfaces are considered. A possible configuration is shown in Fig. 1. The geometrical overlap radius (R_0) is defined as the spherical radius where the channels at the highest latitude just touch each other along a latitude line with channels in adjacent cones touching along longitudinal lines. The current-neutralized ion beams are guided by applied uniform plasma channel currents I_c. The ions are injected into each channel uniformly in cylindrical radius about the channel axis and with a cosine distribution in velocity angles up to a maximum angle θ_m determined by efficient beam transport along the channel.[5,6] The initial beam radius is designated by r_b and the channel radius (equilibrium beam radius) by r_c.

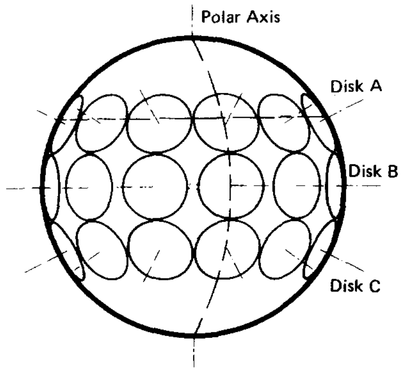

FIG. 1. Example of multi-channel overlap geometry.

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